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PRECISION AIRCRAFT NAVIGATION BASED ON LORAN-C AND DIGITAL ALTIMETRY

by L. F. FEHLNER

THE JOHNS HOPKINS UNIVERSITY ▪ APPLIED PHYSICS LABORATORY

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This report describes a system that can provide precision aircraft navigation based on Loran-C and digital barometric altimetry. The system is expected to be capable of fulfilling the critical accuracy requirements of the terminal area as well as serving the needs of area navigation. A digital barometric altimetry technique is described, which has sufficient accuracy to provide the third dimension for all phases of aircraft flight. Further, several auxiliary functions are described that are made practicable by high-accuracy positioning. These are: on-board determination of the point to release an object for free fall to the ground, a built-in approach and departure system, and automatic direction finding.

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ABSTRACT

This report describes a system that can provide precision aircraft navigation based on Loran-C and digital barometric altimetry. The system is expected to be capable of fulfilling the critical accuracy requirements of the terminal area as well as serving the needs of area navigation. A digital barometric altimetry technique is described, which has sufficient accuracy to provide the third dimension for all phases of aircraft flight. Further, several auxiliary functions are described that are made practicable by high-accuracy positioning. These are on-board determination of the point to release an object for free fall to the ground, a built-in approach and departure system, and automatic direction finding.

PREFACE

This report records the current state of the art in the use of Loran-C for accurate aircraft navigation and for other aircraft functions that are made practicable by accurate navigation. During the past five years the state of the art has been influenced substantially by a number of developments, due principally to the efforts of the author and a number of his associates, namely, Thomas A. McCarty, Thomas W. Jerardi, and Ronald G. Roll. In addition to their valuable contributions throughout the report, Mr. McCarty has specialized in the targeting aspects of the problem and in the understanding of the Loran positioning service; and Mr. Jerardi and Mr. Roll have contributed extensively to the navigation concepts, ballistic solution, and coordinate conversion. In particular, Mr. Jerardi and Mr. Roll are responsible for Appendix B, and Mr. Roll for Appendix C. Without their assistance and the assistance of many others during the last five years, the present report could not have been written. In particular, we wish to express our appreciation to Maurice G. Vincent, William A. Mayhew, Jr., and Donald P. Martini for their valuable guidance and support. Publication of this report was sponsored by the Defense Special Projects Group.

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ILLUSTRATIONS

1. INTRODUCTION

Navigation is perhaps the oldest applied science. Long before the Romans, hardy men of the sea were successfully navigating their ships to distant ports, and camel caravans navigated the featureless deserts. Their problems, however, were a bit different than those of a jet aircraft terminating its flight onto a 200-foot wide runway at 160 knots. To solve these problems and those associated with traffic congestion, navigational aids are required that permit the aircraft to determine its position frequently and precisely.

The Long Range Navigation known as Loran-C is one of many aids to navigation, which are now available to solve present-day problems. Except for a few military applications, very few aircraft use the existing Loran-C service. One purpose of this report is to describe how Loran-C can be used in aircraft for accurate horizontal positioning — in fact, sufficiently accurately to fulfill all horizontal requirements during landing.

Since the earliest days of aircraft, altitude has been estimated from measurements of atmospheric static pressure. The radar altimeter was a natural invention, but this device has found only limited acceptance for a few special functions such as verifying ground clearance. They lack the general utility of a barometric altimeter without which no aircraft flies. Another purpose of this report is to describe the high-accuracy, barometric altimetry techniques that have also been applied in a few military systems.

The final purpose of this report is to describe some auxiliary functions that high-accuracy positioning makes practicable, such as the on-board determination of the point at which to release an object for a free fall to the ground, a built-in approach and departure system, and automatic direction finding.

2. BACKGROUND

A common grid is a system of readily measurable coordinates and time signals, which form a basic reference system, making it possible for all users to make accurate, real-time determinations of present position and epoch time. Since all users of the system refer to a common reference, transfer of coordinates from one user to another incurs no loss of accuracy relative to each other. Instrumentation to recover epoch time makes it feasible to execute critically timed tasks. Lines of position established by Loran are very nearly stationary in space, independent of weather conditions, diurnal effects, foliage density, terrain roughness, or height above terrain. This is not to say that environmental conditions present no problem. Inadequate user equipment or overwhelming noise may make it difficult to obtain a statistically adequate measurement. Precipitation static may cause poor signal-to-noise ratio at an aircraft antenna. How well the user can make measurements under difficult environmental conditions is critically dependent on the design of system components. However, to the extent that the user can make the necessary measurement, the Loran lines of position exist as a stationary grid.

Figure 1 portrays the common grid system. The Loran service provides time differences TDA and TDB, and the pressure transducer provides the pressure P. Clearly, if any of the coordinates of two aircraft are different simultaneously, the aircraft are not in the same place. Conversely, if one aircraft guides to a given set of coordinates, such as TDA, TDB, and P_0 for the touchdown point on a runway, it will indeed be there when its own coordinates are the same as those of the touchdown point. This is a fundamental principle. All navigation, horizontal or vertical, can be done in a relative sense in a common grid system. The result is a simple, opera-

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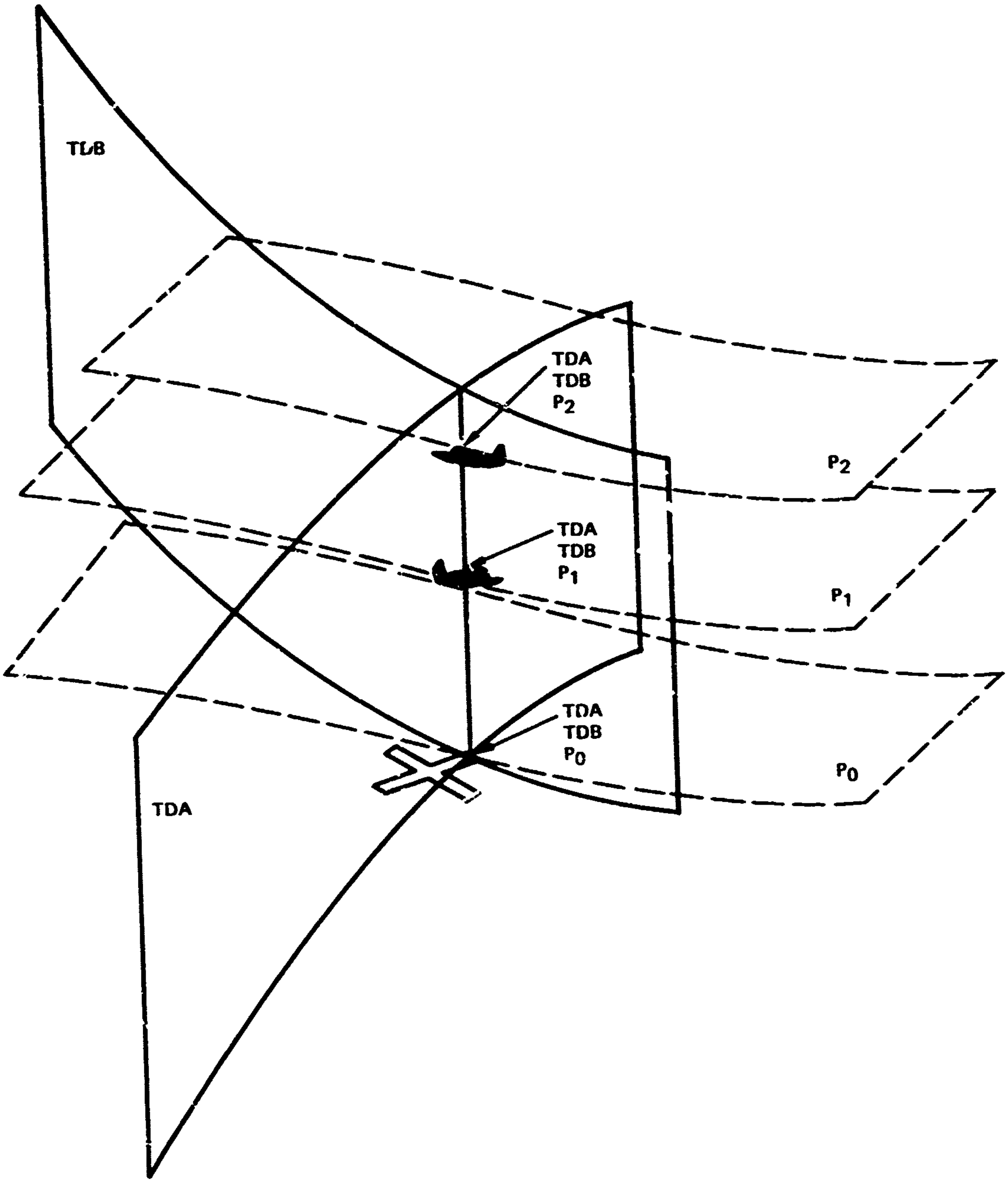

Fig. 1 COMMON GRID SYSTEM

tionally useful navigation technique of sufficient accuracy to satisfy the need in all phases of flight with the possible exception of a soft touchdown under instrument meteorological conditions.

Loran-C radiates, from three or more transmitters, pulsed electrical signals that are very accurately timed and coded for identification. These signals are propagated both as groundwaves and as skywaves reflected from the ionosphere. In the service area, the time relationships that exist among groups of signals depend on where they are observed, and therefore can be used as a measurement of horizontal position.

The standard measurement technique consists of measuring the times of arrival of signals from two transmitters A and B relative to the time of arrival of the signal from a third transmitter, M. These two time differences TDA and TDB identify two hyperbolas that intersect at the receiving antenna, as in Fig. 2. The geometry is tied to the ground in the groundwave service area by taking into account the location of the transmitters and the propagation velocity. For skywave service, the height of the ionosphere also must be taken into account

If all the Loran-C transmitters are controlled by Cesium time standards and synchronized, it is expected that stationary users of the signal within the groundwave service area will be able to synchronize time to 2 μs (3-sigma) and moving users to the order of 5 μs.

The precision of position location available from Loran service is dependent on the precision of the time differences as they exist in the service area. All Loran-C chains are monitored at a site in the service area. The function of the monitor is to compare measured values of time differences to standard values established for the site and to use the differences to control the chain. The best example of this process in operation is in Southeast Asia (SEA). Analysis of extensive data recorded 24 hours

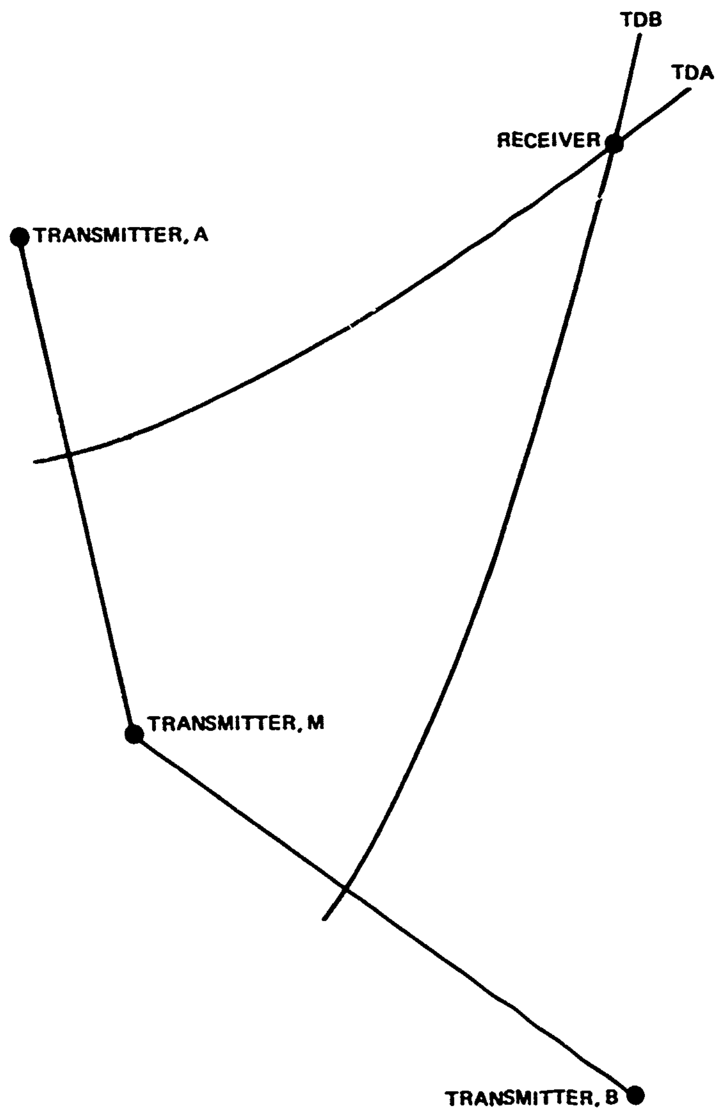

Fig. 2 LORAN EOMETRY

a day during summer, winter, wet and dry seasons at the monitor side at Udorn, Thailand, shows that the mean of the time differences were controlled within 24 nsec of the standard control value, with a standard deviation less than 32 nsec (see Appendix A). From the data of Appendix A, the precision of location on a geometric dilution contour of 1 m/nsec fell between 8 and 40 meters CEP. Improved monitoring systems planned for future Loran chains show promise of control substantially better than is now possible by the manual methods in use in SEA.

Atmospheric static pressure is used universally as a measure of aircraft altitude. The so-called pressure altitude of an aircraft is the ambient static pressure stated in terms of feet derived from the relationships between static pressure and geometric altitude defined by the ICAO Standard Atmosphere. The static pressure at sea level for this atmosphere is 29.92 inches of mercury. If true geometric altitude is desired, it must be estimated by applying corrections to the pressure altitude, which are themselves estimates of the differences between the standard atmosphere and the atmosphere that currently supports the aircraft.

3. THE PROBLEM

The problem is to provide a simple, reliable, and inexpensive means for computing accurate guidance from Point A to Point B along any path, defined either in advance (fixed course) or dynamically in flight (homing); and to produce accurate indications of arrival at specific points. The required accuracy of guidance depends on the function to be performed. It is commonly stated, for example, that the accuracy required for area navigation is far less critical than that required for approaches or departures. This is basically true and could be used as a basis for accuracy criteria in any system in which area navigation aids are different from terminal area aids. Suppose, however, that a single aid could be deployed, which could serve the critical accuracy requirements of the terminal area and could be available to serve the needs of area navigation. Such a system has great utility and economic benefits, since it reverses the trend toward proliferation of navigation aids while at the same time makes it possible to execute high-accuracy functions anywhere. Included among these functions are aerial delivery, aerial survey, traffic control, collision avoidance, and approach and departure at all classes of airfields. The next section describes such a system. Its applicability to traffic control, collision avoidance, and nonairborne uses are covered in Ref. 1.

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4. SYSTEM DESCRIPTION

The system will be described in terms of the aircraft system and its supporting ground system.

AIRCRAFT SYSTEM

The aircraft system is shown as a functional block diagram in Fig. 3.

Initialization

The data required for initialization of the system are of two kinds, shown in Fig. 3 as "Initial Conditions" and "Flight Program." "Initial Conditions" consists of the numerical values of a list of constants that apply for the duration of a single flight. These constants are listed in Table 1.

The second kind of data, "Flight Program," are shown in Table 2. This is organized chronologically in such a way that stepping down the table one line at a time provides all the data necessary for the current leg of the flight. Looking one step ahead provides the data necessary to make the transition from one leg to the next. All of the data of Table 1 could be incorporated in Table 2 if desired, but this would result in substantial repetition.

Table 2 is organized to permit the flight to pass through any number of Loran triads and chains. It also allows any three transmitters to be designated a triad. Most Loran receivers operate only on one group repetition period at a time. Therefore, with these receivers, all three transmitters would have to be members of one chain. Also, most receivers require the master transmitter to be a member of each triad. Neither of these restrictions is necessary in receivers of the future.

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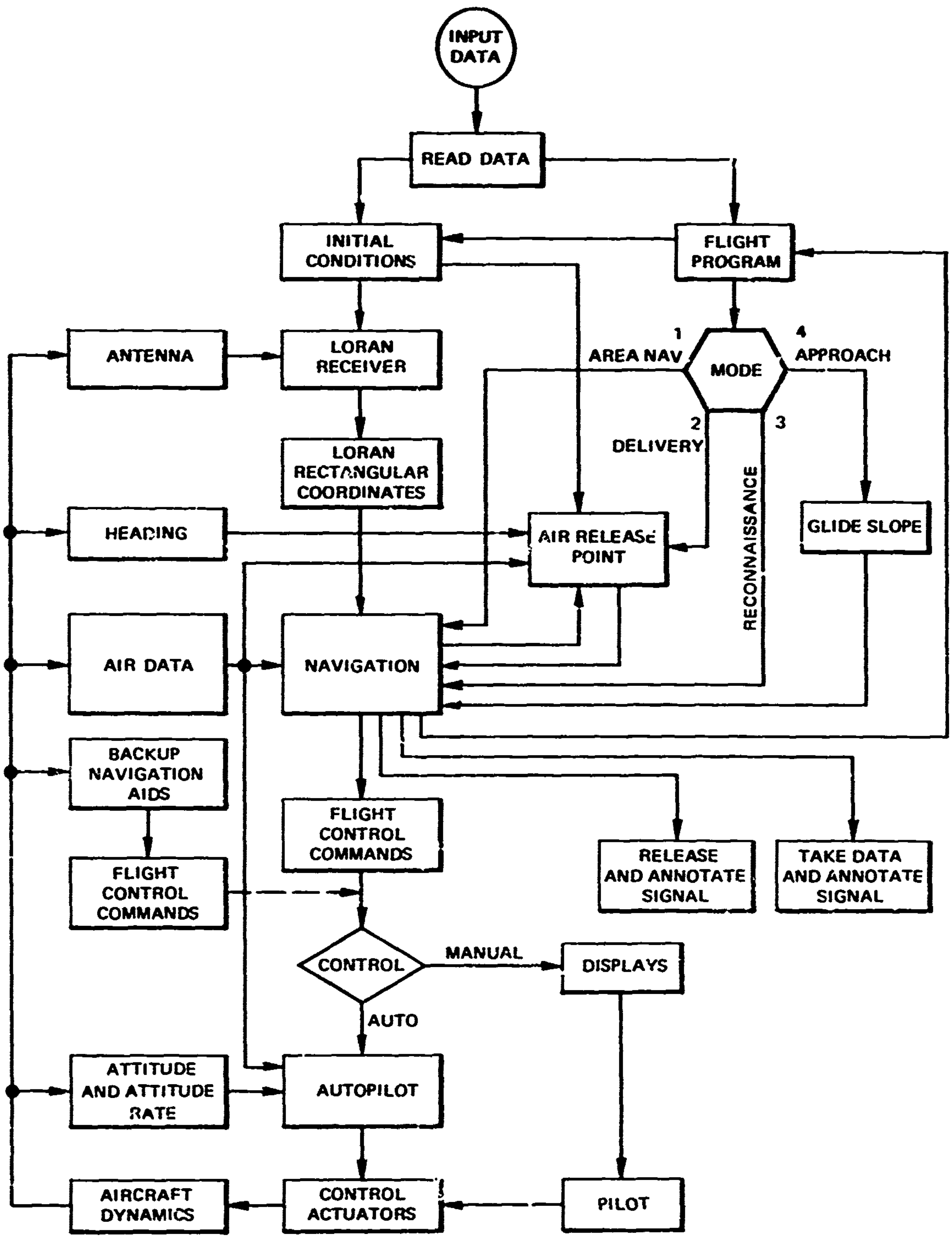

Fig. 3 AIRCRAFT SYSTEM

TABLE 1

CONSTANTS FOR INITIAL CONDITIONS

a. Loran Receiver

P_I	The group repetition period of the I^{th} chain;
$D_{I,J}$	The coding delays for the J^{th} transmitter on the I^{th} chain, J = 1, 2, 3 ... M.

b. Conversion to Loran Rectangular Coordinates

$X_{I,J}, Y_{I,J}$	The Loran rectangular coordinates of the J^{th} transmitter in the I^{th} chain, I = 1, 2 ... N; J = 0, 1, 2 ... M, where J = 0 designates the master;
V	The propagation velocity of the Loran ground wave.

c. Ballistic Data

There are two alternative initial conditions depending on the choice of ballistics algorithm embodied in the avionic computer.

c-1. The simplest algorithm designed for horizontal releases requires the following data:

TR_0, TF_0	The TRail and Time of Fall of the store when released under the planned conditions of release;
TRV, TFV	The partial derivatives of TR and TF with respect to Velocity pressure taken at the planned true air speed at release;
TRH, TFH	The partial derivatives of TR and TF with respect to Height pressure at the planned altitude of release;
VP_0	The desired Velocity Pressure at release
ψ	The compass variation plus deviation.

TABLE 1 (cont'd)

c-2. The more complex algorithm requires the following data:

BI_K	The ballistic index number that identifies store K;
ψ	The compass variation plus deviation.

d. Approach

$\tan \gamma_A$	The tangent of the glide slope for approach;
S_0	The slope of pressure versus altitude at sea level;
S_1	The correction factor to obtain the slope of pressure versus altitude at altitudes above sea level.

e. General

a	Nominal speed of sound;
TCR_{max}	The maximum horizontal angular rate of turn for the aircraft;
$\tan \varphi_{max}$	The tangent of the maximum bank angle of the aircraft.

TABLE 2

FLIGHT PROGRAM

Waypoint Number	Waypoint Location (Present Triad)	Waypoint Location (Next Triad)	TOA	Aircraft Altitude	Mode*	Target Altitude	Chain	Master	Slave A	Slave B	BI**
0	X_0, Y_0	--	T_0	HP_0	1	--	I_0	J_0	J_0	J_0	--
1	X_1, Y_1	X_1', Y_1'	T_1	HP_1	1	--	I_1	J_1	J_1	J_1	--
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
j	X_j, Y_j	--	T_j	HP_j	2	GP_j	I_j	J_j	J_j	J_j	K_j
.	.	.	.	.	.		.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
n	X_n, Y_n	--	T_n	HP_n	4	GP_n	I_n	J_n	J_n	J_n	--

*Possible modes are. 1. Area Navigation, 2. Delivery, 3. Reconnaissance, 4. Approach. One example is listed.

**Example listed is for store K to be delivered to waypoint X_j, Y_j. Flight takes place in two triads with boundary between waypoints 1 and 2.

Table 2 also shows an example of the listings required in three columns for a particular flight. In the "Mode" column, the flight starts in the area navigation mode and proceeds in that mode until waypoint j. Here is entered the designator 2 for delivery mode, indicating that the leg of the flight approaching waypoint j is a delivery leg, and guidance is to be provided to an air release point such that delivery will be made to waypoint j associated with target pressure GP_j. In the "BI" column, a ballistic index number is listed also against waypoint j, indicating the specific store to be released, assuming the more complex ballistic algorithm is used (see Sect. c-2 of Table 1). If the simpler algorithm (Sect. c-1 of Table 1) is used, the "BI" column is not needed. If the designator 3 for reconnaissance were entered, guidance would be to waypoint j and, on arrival, a signal for gathering data would be produced. In the "Location (Next Triad)" column, a pair of coordinates are shown against waypoint 1, which are the coordinates in the next triad, which corresponds to the same point as the coordinates shown in the "Location (Present Triad)" column. When this double listing of coordinates occurs, it indicates that the next leg of the flight beyond waypoint 1 is to be flown on a course from X_1', Y_1' to X_2, Y_2 in the new triad. Finally, in the "Mode" column, designator 4 is listed for approach. As a consequence, the last leg is flown on a glide path to waypoint n and ground pressure GP_n.

Loran Receiver

Refer now, in Fig. 3, to the block "Loran Receiver." The initial conditions for the receiver, given in Table 1, are set to the values required by the flight program, Table 2, and the receiver produces the Loran coordinates, TDA and TDB, of present position and the time rates of change of TDA and TDB. (Details of how Loran receivers produce hyperbolic coordinates and their time rates are beyond the scope of the present effort.)

Loran Rec angular Coordinates

The next block is "Loran Rectangular Coordinates." The function of this block is to convert the Loran hyperbolic coordinates to Loran rectangular coordinates. This is done as follows:

Let

X_M, Y_M	be the coordinates of the Master,
X_A, Y_A	be the coordinates of Slave A,
X_B, Y_B	be the coordinates of Slave B,
X_R, Y_R	be the coordinates of the receiver corresponding to TDA, TDB,
TDA	be the time difference A,
TDB	be the time difference B,
DA	be the absolute emission delay for Slave A, and
DB	be the absolute emission delay for Slave B.

Then, for the current chain and transmitters identified by Table 2,

$X_M, Y_M = X_{I,J}, Y_{I,J}$; J = value for Master,

$X_A, Y_A = X_{I,J}, Y_{I,J}$; J = value for Slave A,

$X_B, Y_B = X_{I,J}, Y_{I,J}$; J = value for Slave B,

DA $= D_{I,J}$; J = value for Slave A,

DB $= D_{I,J}$; J = value for Slave B,

and the following expressions (using self-explanatory dummy variables for clarity) produce the values of X_R, Y_R corresponding to TDA and TDB (see Appendix B):

$$A = Y_M - Y_A$$
$$B = Y_M - Y_B$$
$$C = X_M - X_A$$
$$D = X_M - X_B$$
$$\Delta = A \cdot D - B \cdot C$$
$$\mu_A = V \cdot (TDA - DA)$$
$$\mu_B = V \cdot (TDB - DB)$$
$$E = D \cdot \mu_A$$
$$F = C \cdot \mu_B$$
$$\alpha = 0.5(\mu_A \cdot E - \mu_B \cdot F)/\Delta$$
$$\beta = (E - F)/\Delta$$
$$G = A \cdot \mu_B$$
$$H = B \cdot \mu_A$$
$$\gamma = 0.5(\mu_B \cdot G - \mu_A \cdot H)/\Delta$$
$$\delta = (G - H)/\Delta$$
$$\xi = (Y_M - \alpha)^2 + (X_M - \gamma)^2$$
$$\zeta = 2(\alpha\beta - Y_M\beta + \gamma\delta - X_M\delta)$$
$$\eta = \beta^2 + \delta^2 - 1$$

$$(\theta_M)_{1,2} = \frac{-\zeta \pm \sqrt{\zeta^2 - 4\xi\eta}}{2\eta}.$$

Negative solutions for θ_M are not real. Therefore, these can be eliminated by a simple sign check. However, there are regions in the service area where two positive values are obtained for θ_M. In this case, a priori knowledge must be used to select the correct solution as follows:

Let

X_0, Y_0 be the coordinates of the receiver position at the beginning of a flight (see Table 2).

Then

$$(\theta_M)_0 = \sqrt{(X_M - X_0)^2 + (Y_M - Y_0)^2} .$$

Now let

$$(\Delta\theta_M)_1 = (\theta_M)_0 - (\theta_M)_1 ,$$

and

$$(\Delta\theta_M)_2 = (\theta_M)_0 - (\theta_M)_2 .$$

Taking absolute values, if

$$|(\Delta\theta_M)_1| > |(\Delta\theta_M)_2| ,$$

let

$$\theta_M = (\theta_M)_2 .$$

If

$$|(\Delta\theta_M)_1| < |(\Delta\theta_M)_2| ,$$

let

$$\theta_M = (\theta_M)_1 .$$

If

$$|(\Delta\theta_M)_1| = |(\Delta\theta_M)_2| ,$$

let

$$\theta_M = (\theta_M)_1 ,$$

if previous θ_M's had been $(\theta_M)_2$.

Let

$$\theta_M = (\theta_M)_2 ,$$

if previous θ_M's had been $(\theta_M)_1$.

Then let

$$(\theta_M)_0 = \theta_M ,$$

and solve for

$$X_R = \gamma + \delta \cdot \theta_M ,$$

$$Y_R = \alpha + \beta \cdot \theta_M .$$

The velocity components of the receiver, $\dot{X}_R$ and $\dot{Y}_R$ are obtained as follows:

Let

$$a = \frac{X_R - X_A}{\theta_M + \mu_A} - \frac{X_R - X_M}{\theta_M} ,$$

$$b = \frac{Y_R - Y_A}{\theta_M + \mu_A} - \frac{Y_R - Y_M}{\theta_M} ,$$

$$c = \frac{X_R - X_B}{\theta_M + u_B} - \frac{X_R - X_M}{\theta_M} ,$$

$$d = \frac{Y_R - Y_B}{\theta_M - \mu_B} - \frac{Y_R - Y_M}{\theta_M} ,$$

and

$$AA = \frac{d}{ad - bc} ,$$

$$BB = \frac{-b}{ad - bc} ,$$

$$CC = \frac{-a}{ad - bc} ,$$

$$DD = \frac{c}{ab - bc} .$$

Then

$$\dot{X}_R = V(AA \cdot \dot{T}DA + BB \cdot \dot{T}DB)$$

$$\dot{Y}_R = V(CC \cdot \dot{T}DA + DD \cdot \dot{T}DB)$$

where

$\dot{T}DA$ and $\dot{T}DB$ are the time rates of change of TDA and TDB.

The logic given above for selecting the sign of the square root in the solution for θ_M is involved with an indeterminacy whenever $(\theta_M)_1 = (\theta_M)_2$. This equality only exists along base line extensions. (In a triad, there are three base lines and six baseline extensions. An extension is a line drawn from a transmitter in a direction opposite to the direction of any other transmitter.) If the logic given above is used when $(\theta_M)_1$ and $(\theta_M)_2$ are equal, the indeterminacy is resolved for cases where the baseline extension is being crossed at a reasonable angle. However, if a sufficient length of time is spent continuously in close proximity to the baseline extension, the navigation computer will lose track of which way it should leave the baseline extension. This problem can be resolved only with additional a priori information. The logic required to resolve this complication will not be developed in this paper because Loran along baseline extensions is known to be poor for other reasons. The preferred

resolution of the problem is to place operational constraints on aircraft and Loran chain design so that aircraft never have to fly parallel and close to baseline extensions.

Navigation

Refer now to the block labeled "Navigation" in Fig. 3. The inputs from "Loran Rectangular Coordinates" are a continuous succession of the receiver coordinates X_R and Y_R and its velocity components $\dot{X}_R$ and $\dot{Y}_R$. Another input, prior to the start of each leg independent of mode, is the waypoint information from the "Flight Program" of Table 2. In the case of the delivery mode, the upcoming waypoint is adjusted to the coordinates of the release point.

a. Fixed Course. The navigation problem is "getting from waypoint j to waypoint j + 1" for any value of j from 0 to n - 1. Figure 4 displays this general situation with the desired guidance data, Cross Track Error (CTE), and Along Track Distance to go (ATD). This navigation mode is called "Fixed Course."

Let us denote $X_{j+1} - X_j$ as ΔX_j and $Y_{j+1} - Y_j$ as ΔY_j, as in Fig. 4. The true course from j to j+1 is TC_j, and the following relations hold

$$\sin TC_j = \frac{\Delta X_j}{\sqrt{\Delta X_j^2 + \Delta Y_j^2}} ;$$

$$\cos TC_j = \frac{\Delta Y_j}{\sqrt{\Delta X_j^2 + \Delta Y_j^2}} .$$

Now, the current position of the receiver is X_R, Y_R, so that

$$\Delta e_{j+1} = X_{j+1} - X_R ;$$

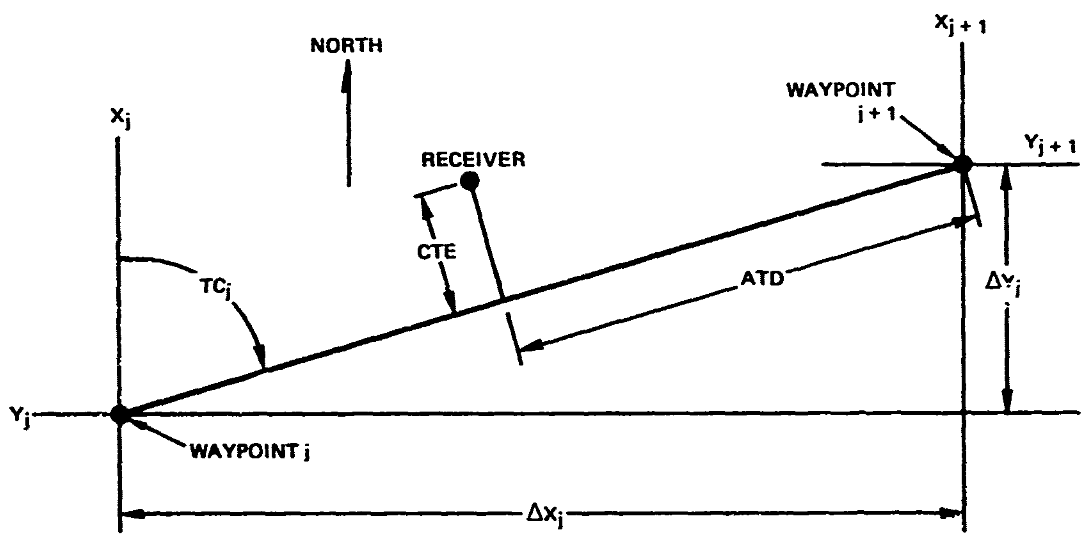

Fig. 4 HORIZONTAL FIXED COURSE GEOMETRY

$$\Delta n_{j+1} = Y_{j+1} - Y_R \ ;$$

where Δe_{j+1} and Δn_{j+1} are the east and north positions of the receiver with respect to the j + 1th waypoint. The steering signals are now given by

$$CTE_{j+1} = \Delta e_{j+1} \cos TC_j - \Delta n_{j+1} \sin TC_j \ ,$$

$$ATD_{j+1} = \Delta e_{j+1} \sin TC_j + \Delta n_{j+1} \cos TC_j \ ,$$

$$CTER_{j+1} = -\dot{X}_R \cos TC_j + \dot{Y}_R \sin TC_j \ .$$

$$ATDR_{j+1} = -\dot{X}_R \sin TC_j - \dot{Y}_R \cos TC_j \ .$$

where CTER and ATDR are time rates of change of CTE and ATD. If required,

$$HE = \tan^{-1} \frac{CTE}{ATD} \ ,$$

where HE is the heading error.

b. Homing. Fixed course guidance has been described above. Another option is "Homing" guidance. In this case, Fig. 5 pertains, and

$$\sin TC_j = \frac{\Delta e_{j+1}}{\sqrt{\Delta e_{j+1}^2 + \Delta n_{j+1}^2}} \ ,$$

$$\cos TC_j = \frac{\Delta n_{j+1}}{\sqrt{\Delta e_{j+1}^2 + \Delta n_{j+1}^2}} \ .$$

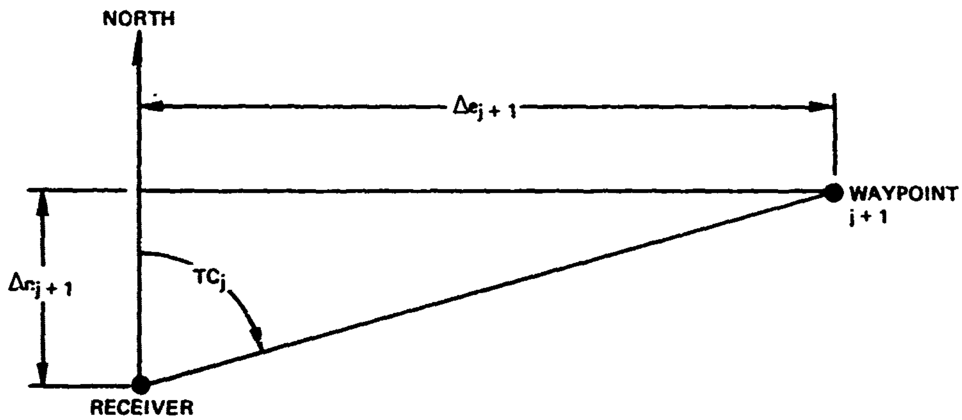

Fig. 5 HORIZONTA HOMING GUIDANCE GEOMETRY

The ADF function (Automatic Direction Finding) is fulfilled by taking

$$TC_j = \sin^{-1} \frac{\Delta e_{j+1}}{\sqrt{\Delta e_{j+1}^2 + \Delta n_{j+1}^2}} .$$

The steering signals are now given by:

$$CTE_{j+1} = 0 ,$$

$$ATD_{j+1} = \Delta e_{j+1} \sin TC_j + \Delta n_{j+1} \cos TC_j ,$$

$$CTER_{j+1} = -\dot{X}_R \cos TC_j + \dot{Y}_R \sin TC_j ,$$

$$ATDR_{j+1} = -\dot{X}_R \sin TC_j - \dot{Y}_R \cos TC_j .$$

c. Turns. If a transition from one flight leg to the next is not initiated until the upcoming waypoint is reached, the turn will result in considerable overshoot. The point to initiate the turn in advance of the upcoming waypoint is determined as follows: Aircraft are often steered by executing standard rate turns unless the demands on the aircraft require bank angles greater than a stated maximum, in which case the maximum bank angle is executed. Thus,

$$TCR \leq TCR_{max} ,$$

and

$$\varphi \leq \varphi_{max} ,$$

where TCR is the time rate of change of course and φ is the angle of bank in a steady-state turn. First find

$$TAS = Ma ,$$

$$R_1 = \frac{TAS}{TCR_{max}} .$$

Then find

$$R_2 = \frac{TAS^2}{g \tan \varphi_{max}} ,$$

where

TAS is the True Air Speed,

a is the speed of sound,

R is the radius of turn, and

g is acceleration of gravity.

If $R_1 \leq R_2$, execute the turn at a point determined by R_1. If $R_1 > R_2$, execute the turn at a point determined by R_2. Now let R be either R_1 or R_2, depending on the above conditions, and refer to Fig. 6.

Let

$$\Delta TC = TC_{j+1} - TC_j ,$$

and

$$\Delta ATD = R \tan \frac{\Delta TC}{2} .$$

Depending on the sign of ΔTC, a right or left turn is executed at the Turning Point, TP, that is, when ATD = ΔATD. There are four options for making the turn. The first consists of flying the standard turn associated with either TC_{max} or φ_{max} until such time as the next leg of flight progresses to ΔATD from the starting waypoint, at which time the standard turn is terminated and guidance resumed according to CTE indications. The second option is to change at TP from Fixed Course to Homing on waypoint j+2, then change back to Fixed Course when

$$(CTER_{j+2})_{Homing} = 0 \pm (CTER)_{Tolerance} .$$

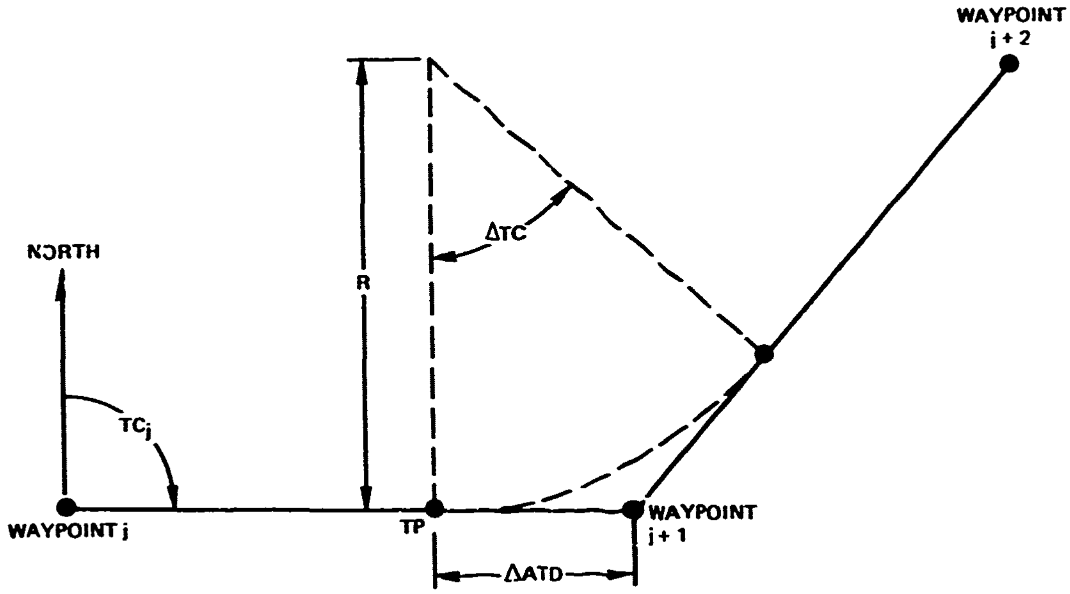

Fig. 6 TURN GEOMETRY

Having passed waypoint j+1, the Fixed Course guidance will be along the leg j+1 to j+2. The third option is to change to Fixed Course guidance along leg j+1 to j+2 at TP, at which time substantial CTE and CTER will exist, indicating the correct guidance to the desired course. This technique can produce very accurate ground tracks in a turn by spacing the waypoints very closely during the turn, and it is the simplest to execute. If guidance is required along a continuous curved path, a fourth option would be to compute on-board the geometry of the curve desired for the turn.

d. Vertical Guidance. Another input to the "Navigation" block is atmospheric static pressure, i.e., the height pressure of the aircraft HP, and the height pressure time rate HPR. Also, an input from the Flight Program is the height pressure associated with the waypoints. There are basically two ways to handle vertical guidance in the area navigation mode. The first way is to produce continuous altitude indications analogous to the continuous horizontal guidance described above. The second way is to produce incremental vertical indications between waypoints.

Figure 7 displays the general situation with the desired vertical guidance data, Height Pressure Error (HPE) and Height Pressure Error Rate (HPER). Figure 7 is the situation in a vertical plane passed through the waypoints of Fig. 4. Let us denote $HP_{j+1} - HP_j$ as ΔHP_j. The desired climb path angle from j to j+1 is γ, and the following relation holds:

$$K \tan\gamma = \frac{\Delta HP_j}{\sqrt{\Delta X_j^2 + \Delta Y_j^2}} .$$

Now, the current position of the pitot is HP, so that

$$HPE_{j+1} = HP_{j+1} - ATD_{j+1} K \tan\gamma - HP .$$

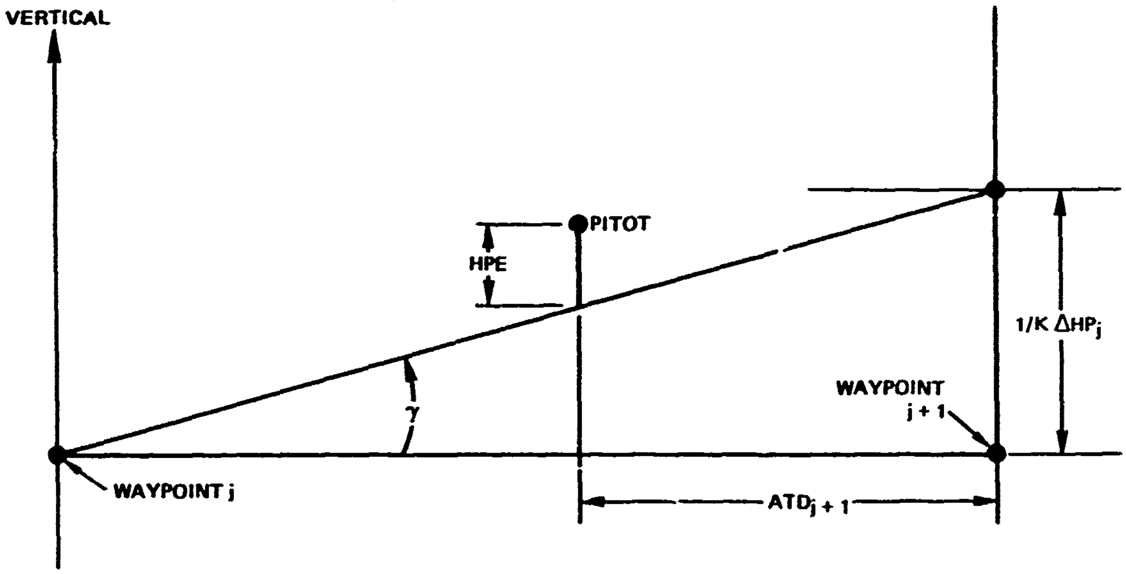

Fig. 7 VERTICAL GUIDANCE GEOMETRY

Similarly

$$HPER_{j+1} = ATDR_{j+1}\ K \tan\gamma - HPR.$$

This relation holds for all j, and does not require any trigonometric routines. Another set of outputs of the "Navigation" block is height pressure error and height pressure error rate. In flight planning, restrictions must be placed on γ so that the demands on the aircraft do not exceed its capability.

Referring again to Fig. 7, the incremental vertical guidance techniques consist of letting

$$HPE_{j+1} = HP_{j+1} - HP\ .$$

Then, depending on the sign of HPE, a standard climb or glide is executed until $HPE_{j+1} = 0$. At this point, the climb or glide is terminated and level flight achieved, such that HPE_{j+1} continues to be zero. In this case, during flight planning the waypoints could be placed so close together that the aircraft cannot reach the desired altitude by the time it begins the next leg. This is of no consequence unless being at altitude throughout the leg j+1 to j+2 is important to the mission.

e. Arrival Time. Under some circumstances it may be important to arrive at waypoints at specified times. If this is the case, the required ground speed along each leg of the flight is determined as follows:

$$ATDRE_{j+1} = \frac{ATD_{j+1}}{T_{j+1} - T_R} - ATDR_{j+1}\ ,$$

where

T_{j+1}	is the required Time Of Arrival (TOA) at waypoint j+1,
T_R	is time at the receiver, and
ATDRE	is the along-track distance rate error.

During mission planning, the time of arrival at each waypoint must be related to the length of the leg so that the speed capability of the aircraft is not exceeded, considering the influence of winds aloft.

Air Data

The vertical positioning of the aircraft and the horizontal location of an air release point are dependent on the accuracy of the air data. Two pressure measurements are required, ram and static. The measurement of true ram pressure is not difficult since a true value is obtained when the air is suddenly brought to rest anywhere outside the boundary layer. However, very accurate measurements of true static pressure are virtually impossible because of the potential flow interference effects. These interference effects must be accounted for through accurate calibration of the static pressure measurements for the specific location of the static orifices, for the operational speed range, and for operationally important flight configurations, e.g., flaps versus no flaps.

Let

P_R	be the ram pressure,
P_S	be the static pressure measurement,
ΔP	be the static pressure interference effect,
HP	be the true static pressure or Height Pressure,
HPR	be the Height Pressure time Rate of change
VP	be the velocity pressure,
t	be the time,
M	be the Mach number, and
γ	be the ratio of the specific heats for air.

Then, the desired outputs of the "Air Data" block are

$$HP = P_S - \Delta P ,$$

$$HPR = \frac{dHP}{dt} ,$$

$$VP = P_R - HP ,$$

$$M = \sqrt{\frac{2}{\gamma} \frac{VP}{HP}} .$$

There are two options. If the geometric altitude of a specific type of aircraft does not have to be accurately controlled, the output of the "Air Data" block can be "raw" data, i.e., $\Delta P = 0$. In this case, the effects of ΔP can be accounted for in the ground system. Under the other option, the "Air Data" block output is as accurate as the calibration, i.e., ΔP is calculated as a function of Mach number and aircraft trim condition. In this case, the ground system ignores the effects of ΔP.

Air Release Point

The path of an unguided object released from an aircraft is dependent on its aerodynamic characteristics and the initial conditions at release. To launch a specific object on a predicted path, the initial conditions must be controlled to specific values; and to simplify this task, releases are usually made under steady-state conditions. The following computation of air release point applies to steady conditions of true air speed and vertical velocity, i.e., the only acceleration acting on the aircraft is gravity.

Let

TAS be the True Air Speed,

TH be the True Heading,

TF be the Time of Fall,

GS be the Ground Speed,

TR be the TRail,

BA be the Ballistic Altitude,

DR be the DRift Angle,

TC be the True Course,

CT be the Cross Trail,

AR be the Actual Range,

VV be the Vertical Velocity,

BI be the Ballistic Index number,

BR be the Ballistic Range, and

ψ be the compass variation plus deviation.

Then, referring to Fig. 8,

$$TH = H + \psi,$$

$$DR = TC - TH,$$

$$CT = TR \sin DR,$$

$$AR = TF \cdot GS - TR \cos DR,$$

where H is the compass Heading. Table 2 indicates that the target horizontal coordinates are given as one of the waypoints identified by "2" for delivery mode. When this designation occurs, the guidance for that leg of the flight is computed by:

$$CTE_R = CTE_{j+1} + CT,$$
$$ATD_R = ATD_{j+1} - AR,$$

where CTE_R and ATD_R are with respect to the air Release point.

As indicated in Table 1, there are two options for the ballistic solution. The first requires the aircraft vertical velocity to be zero, and it is linearized over limited deviations of altitude and air speed from the values specified by Tables 1 and 2. The vertical and horizontal components of ejection velocity are accounted for by TR_0 and TF_0. The following equations apply:

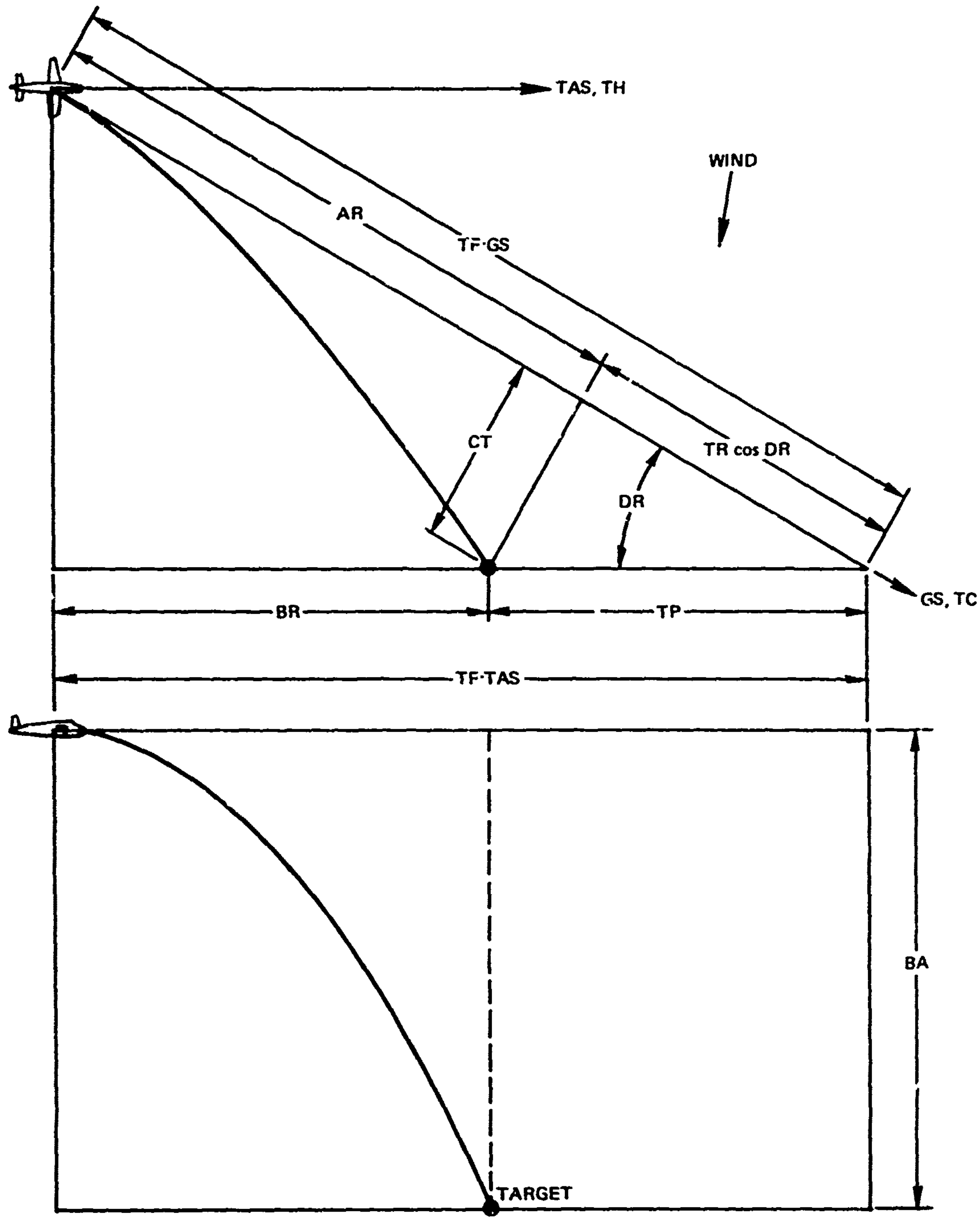

Fig. 8 BALLISTIC SOLUTION

$$\Delta VP = VP_0 - VP,$$
$$\Delta HP = HP_{j+1} - HP,$$
$$TR = TR_0 + TRV \cdot \Delta VP + TRH \cdot \Delta HP,$$
$$TF = TF_0 + TFV \cdot \Delta VP + TFH \cdot \Delta HP,$$

where TRV, TFV are the derivatives of TR and TF with respect to Velocity pressure taken at the planned true air speed at release; and

TRH, TFH are the derivatives of TR and TF with respect to Height pressure taken at the planned altitude of release.

The second option requires:

$$BP = GP_{j+1} - HP,$$

where BP is the Ballistic Pressure height above ground. Ballistic range and time of fall are determined as

$$TR = f_1 (VP, HPR, BP, BI),$$
$$TF = f_2(VP, HPR, BP, BI),$$

where the functions f_1 and f_2 are arbitrary algebraic expressions that represent the ballistic tables for any object released with specified ejection velocity within the flight regime of the aircraft under steady-state conditions. The vertical velocity effect can be accounted for either as a rate of change of pressure HPR from Air Data or as actual vertical velocity converted to HPR in aircraft equipped with suitable inertial components. If the "Air Data" block of Fig. 3 removes the effect of ΔP, the coefficients of the equations for TR and TF reflect "true" values. If ΔP effect is not removed in the aircraft, the coefficients must reflect the calibration, and then the equations for TR and TF apply only to that aircraft.

Glide Slope

Refer to Fig. 9 for the approach geometry where γ_A is the desired glide slope angle and ΔH_n is the desired incremental height above waypoint n located at the touchdown point. Now proceed as follows:

$$\Delta H_n = ATD_n \tan\gamma_A ,$$
$$HP_n = GP_n + [S_0 + S_1(P_S - GP_n)]\,\Delta H_n$$

and

$$HPE_n = HP_n - HP ,$$

where

S_0 is the Slope defined as the derivative of atmospheric pressure with respect to height at sea level,

S_1 is a correction factor for determining the Slope at a height equivalent to GP, and

P_S is the sea level atmospheric pressure of the standard atmosphere.

If values of S_0 and S_1 are determined from the ICAO standard atmosphere using inches of mercury and feet of altitude, $S_0 = -1.05 \times 10^{-3}$ and $S_1 = 2.58 \times 10^{-5}$.

Aircraft Control

The output of the "Navigation" block of Fig. 3 is Cross Track Error (CTE), Cross Track Error Rate (CTER), Height Pressure Error (HPE), and Height Pressure Error Rate (HPER). These outputs must be transformed into control commands to either an autopilot or pilot. In the case of autopilot control, the commands are usually accelerations that are obtained from equations of the form

$$C_1 = A_1(CTE) + B_1(CTER),$$
$$C_2 = A_2(HPE) + B_2(HPER),$$

where

$C_{1,2}$ are the acceleration commands, and

$A_{1,2}$ and $B_{1,2}$ are coefficients that are selected to produce the desired performance.

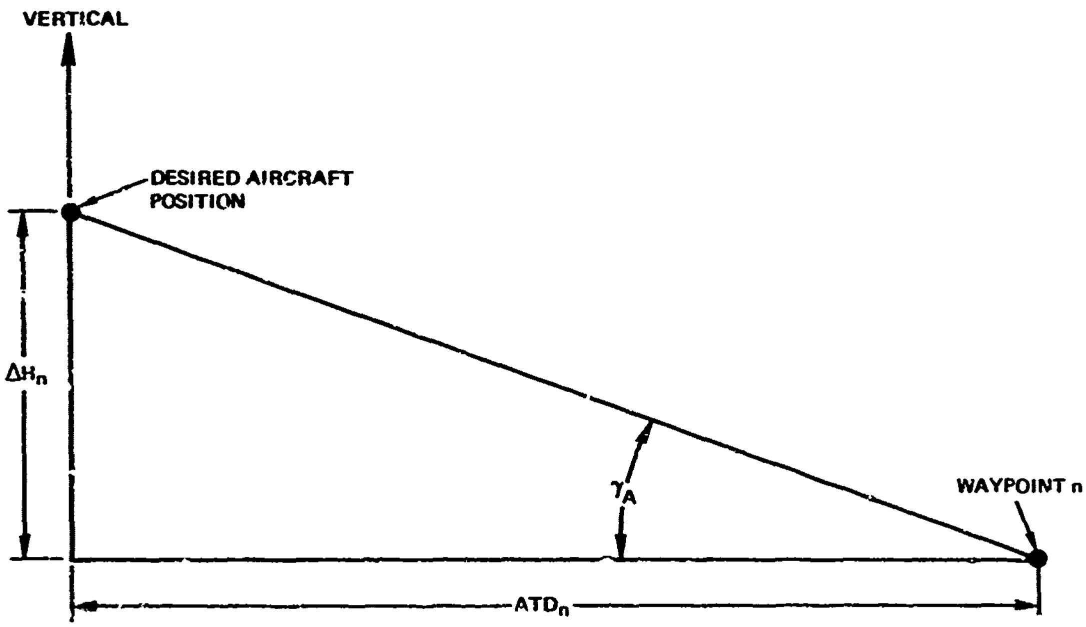

Fig. 9 APPROACH GEOMETRY

In the case of control by the pilot, the commands are displayed on the bank and pitch steering bars of the Attitude Director Indicator (ADI), and CTE is displayed by the Horizontal Situation Indicator (HSI). In either case, the desired "Aircraft Dynamics" are obtained through the control actuators. The feedback loops to Antenna location, Heading, Air Data, and possibly Autopilot characteristics are closed through the aerodynamics.

In case of failure of the primary navigation system, back-up navigation aids may be available. If they are, outputs must be transformed to the same type of flight control commands that the autopilot or cockpit displays receive from the primary system. Their commands would be introduced at the point indicated by the dashed line in Fig. 3. The flight program, initialization, and algebraic description of backup systems are beyond the scope of this paper.

Mode Characteristics

When the aircraft is operating in any of the modes, the system continuously produces guidance signals to fly the flight profile defined by the Flight Program, and signals are produced at each turning point to sequence the "Flight Program" to the next leg of flight. In addition, when the aircraft is operating in the delivery mode, a release signal is produced, and pertinent data are frozen in the navigation computer at the computed air release point. The frozen data are then recorded. When operating in the reconnaissance mode, signals are produced to initiate and terminate the acquisition of data at a waypoint identified by the flight program. Also, appropriate flight data are frozen in the navigation computer and recorded.

GROUND SYSTEM

The ground system required to produce flight programs is shown as a functional block diagram in Fig. 10. Basic data required for target and waypoint coordinates

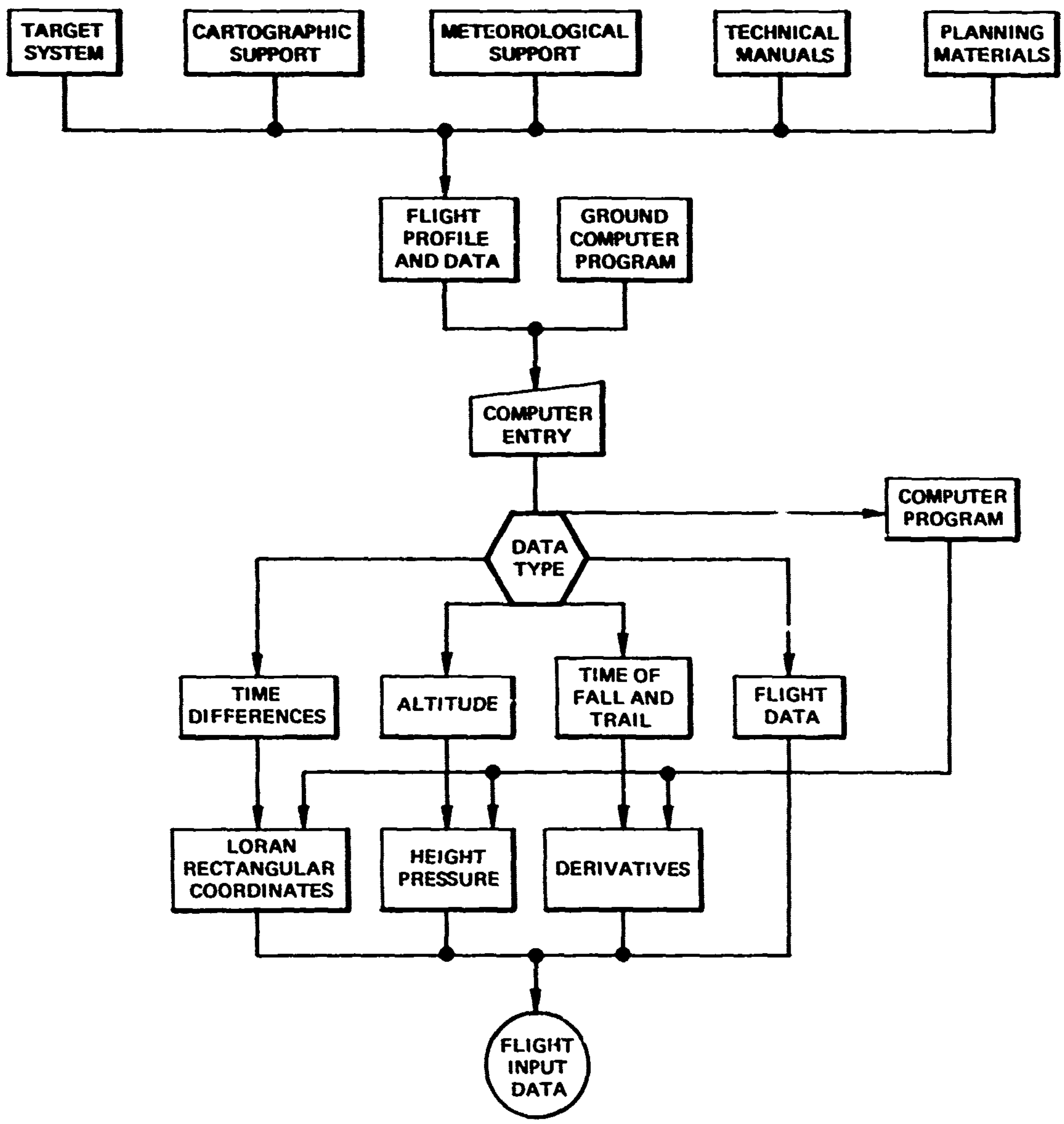

Fig. 10 GROUND SYSTEM

are obtained from the two functional blocks "Target System" and "Cartographic Support." The distinction is that the "Target System" function is to produce the coordinates of specific points with the highest accuracy that is practicable to the aircraft system operating under critical guidance conditions. The "Cartographic Support" function is to produce cartographic materials that allow operators to determine coordinates of less accuracy than the "Targeting System" but of sufficient accuracy for noncritical guidance conditions. "Meteorological Support" provides the information required to convert altitude to height pressure. "Technical Manuals" and "Planning Materials" provide instructions, ballistic data, and tabulation forms for specifying the flight profile and other flight data.

Target System

The basic aircraft reference system depends on Loran time differences and height pressure. Basic targeting systems depend on geodetic coordinates and geometric altitude above sea level. Therefore, a coordinate transformation is required to properly interface the two systems.

In the past the classical transformation has been used, which depends for accuracy on accurate determinations of geodetic distances and propagation velocities. Geodetic distances depend in turn on mathematical models of the earth, and propagation velocities depend on detailed knowledge of geology and its influence on earth conductivity. Experience has shown that, in spite of its complexity, the accuracy of the classical transformation does not fully exploit the accuracy potential of Loran-C without empirical correction factors derived from calibration data. Analysis of errors made by the classical transformation shows that the significant errors are attributable to imperfections of the propagation model. In view of these imperfections and the necessity to calibrate, it is not necessary to continue the use of sophisticated geodetic and propagation models for purposes other than research. The

desired transformation from hyperbolic to geodetic coordinates, or the inverse, is fully realized by the simplest mathematical relationship that preserves the accuracy inherent in the calibration. The following expressions capitalize on this concept and greatly reduce the computer burden associated with the transformation function.

Assume that calibration data are gathered at distinctive geographic features whose geodetic coordinates are accurately known. These can be any standard set, the most common being Longitude/Latitude, and "UTM" (Universal Transverse Mercator). Using UTM as an example, the calibration data are a collection of two pairs of coordinates, UTM and hyperbolic, both of which apply to the same geographic feature. The UTM coordinates E and N are known from prior mapping and the hyperbolic coordinates TDA and TDB are measured. Now let

$$TDA = A_0 + A_1 E + A_2 N + A_3 E \cdot N + A_4 E^2 + A_5 N^2 ,$$

$$TDB = B_0 + B_1 E + B_2 N + B_3 E \cdot N + B_4 E^2 + B_5 N^2 ,$$

and find the values of the A and B coefficients by a least square curve fit process to the calibration data. Substitution of target coordinates E_T and N_T in these equations produces values of TDA and TDB with an accuracy that is at least as good as the calibration data. In cases where redundant data are obtained, the accuracy of TDA and TDB enjoys a statistical improvement.

The targeting system also produces target altitude as geometric height above sea level. Conversion to height pressure is not made by the "Targeting System" because this conversion is most accurate if made with respect to the atmospheric pressure distribution prevailing during the time of the flight in the target area. Due to temporal changes in the atmosphere, this is best deferred as long as possible prior to takeoff or, if the system is so designed in advance, the conversion could be made after takeoff and communicated to the aircraft.

Cartographic Support

For general operational utility, the inverse of the above equations for TDA and TDB can be used to draw a grid of TDA and TDB on the standard maps normally used for laying out flight plans. The accuracy of coordinate transformation accomplished in this manner is degraded somewhat by the necessity to visually recall the data from the maps, and by the fact that maps never represent the current geographic situation. However, such graphics satisfactorily fulfill the need for noncritical waypoint locations and general flight planning.

Meteorological Support

The function of "Meteorological Support" is to supply the atmospheric data required to convert altitude to height pressure. Consistent with the equations shown below under "Height Pressure," the data required are the sea level pressure in the target areas (or any area where vertical aircraft control is critical) and an indication of the prevailing equations of state for the area of operations; for example, the equations for any one of the standard atmospheres of Ref. 2.

Technical Manuals

The required manuals are the standard types that describe the system and how to operate it. The following paragraphs describe that part of the manuals' content that is not obvious from other sections of this report.

In the case of the simpler ballistic solution (see Table 1), the manuals must include ballistic tables that list Trail and Time of Fall versus Velocity Pressure and Height Pressure for specific free-fall objects and dispensers. In the case of the more complex ballistic solution (see Table 1), the manuals include instead, the values of the Ballistic Index for specific free-fall objects. The

manuals must also include the Loran Rectangular Coordinates of the Loran transmitters and other basic chain information.

The values of Time of Fall and Trail as functions of Velocity and Height Pressure are related to the more conventional Time of Fall and Ballistic Range as functions of True Air Speed and Altitude, as follows:

At any position in the atmosphere,

HP is the true static pressure, i.e., Height Pressure,

P_R is the ram pressure,

a is the velocity of sound,

TAS is the true air speed, and

M is the Mach number.

Also,

GP is the static pressure at the target point on the ground.

The two independent variables to be used are the Ballistic Pressure BP, defined as (GP - HP) and the Velocity Pressure VP, defined as (P_R - HP) (see sections on Air Data and Air Release Point).

In isentropic flow,

$$\frac{P_R}{HP} = \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{\frac{\gamma}{\gamma-1}},$$

$$= \left(1 + 0.2\,\frac{TAS^2}{a^2}\right)^{3.5} \quad \text{for air,}$$

and

$$VP = HP\left[\left(1 + 0.2\frac{TAS^2}{a^2}\right)^{3.5} - 1\right].$$

These equations are for use with true Air Data. If the reconstituted ballistic tables are prepared for use with raw "Air Data," i.e., not corrected for ΔP by calibration (see section on "Air Data" under "Aircraft Systems"), $HP + \Delta P$ must be substituted for HP, and $VP - \Delta P$ for VP in the above equations. If TF is the time of fall and BR the ballistic range, then the trail, TR, is given by:

$$TR = TAS \cdot TF - BR.$$

From the standard level-release tables, one may obtain TF and BR in terms of three parameters: (a) true air speed, (b) altitude above the target, and (c) target altitude. Height pressure and sound velocity as a function of altitude may be obtained from Ref. 2. Using the above equations, the ballistic tables can be reconstituted in terms of VP and HP (for various values of GP if variations due to GP are significant). Experience has shown that ballistic data in terms of VP and HP are relatively insensitive to GP. They are also relatively insensitive to mistakes made in the selection of the correct atmospheric model from Ref. 2. It is suggested that the atmospheric model used be chosen as representative of the expected area of operations.

The Technical Manuals must include pertinent data that define the geometry of the Loran chains. These data are the group repetition period P_I; the coding delays $D_{I,J}$; the Loran rectangular coordinates of the transmitters $X_{I,J}$, $Y_{I,J}$; and the propagation velocity (see Table 1). P_I and $D_{I,J}$ can be taken directly from standard publications on existing Loran chains, but $X_{I,J}$, $Y_{I,J}$, and V must be determined from the published longitudes and latitudes of the transmitters and the associated coding delays. The method for doing this is given in Appendix C.

Planning Materials

In addition to standard flight planning materials, forms for chronologically organizing the ground computer input are required. Each entry must be addressed to enable the ground computer to process the input data and organize the output consistent with Tables 1 and 2. Properly executed forms will be referred to as the "Flight Program."

Flight Profile and Data

The result of flight planning is the definition of the flight profile and the associated data. Data recorded on flight program forms described under "Planning Materials" constitute this definition. These data must be consistent with Tables 1 and 2, except that the horizontal coordinates are given in time differences, and altitude in geometric height above sea level. The ground computer converts these inputs to the forms required by Tables 1 and 2. In addition, the input data must include initialization data for the ground computer subroutines. These data are the sea level pressure and the coefficients of the equation of state of the atmosphere. See the section on "Height Pressure" below.

Ground Computer

The function of the ground computer is to convert flight planning parameters to the required avionic parameters. This function is required because the flight planning parameters that are convenient to manipulate manually burden the avionic computer unduly if they are used for navigating the aircraft. The ground computer program must recognize a library of codes associated with the data listed in the Flight Program so that it can properly process each type of input according to "Data Type." It must also include the three subroutines required for "Loran Rectangular Coordinates," "Height Pressure," and "Derivatives." Furthermore, the output format must be consistent with the input format of the "Aircraft System."

Loran Rectangular Coordinates

The inputs to the functional blocks, "Loran Rectangular Coordinates" of Fig. 10, are the time difference coordinates of waypoints, target points, and touch-down points as required to specify the flight profile. The function of this block is to convert the time differences to X_j, Y_j coordinates of Table 2. This is done by a subroutine of the ground computer program. This must be done in exactly the same way as prescribed for the Aircraft System. (See the "Loran Rectangular Coordinate" section under "Aircraft System" for the required algebra.)

Height Pressure

The inputs to the functional block "Height Pressure" are the geometric altitudes above sea level of waypoints, target points, or touch-down points as required. The function of "Height Pressure" is to convert geometric altitude to the HP_j or GP_j coordinates of Table 2 consistent with present mean atmospheric conditions. This is done as follows:

$$HP_j \text{ or } GP_j = P_0(1 - aH)^b ,$$

where

P_0 is the current value of atmospheric pressure at sea level in the area of operation,

a and b are coefficients which define the equations of state of the atmosphere, and

H is the geometric height above sea level of waypoints or ground points.

Sea level pressure and the two coefficients must be entered into the ground computer along with other flight data. Their values are obtained from "Meteorological Support."

As an example, if altitudes were given in feet, and if the latitudes of flight operations corresponded to those where the ICAO standard atmosphere applies, the value of coefficient a is 6.875×10^{-6}, and of b is 5.256. A study of the standard atmospheres of Ref. 2 indicates that the values of a and b are not very sensitive to latitude. It is expected, therefore, that the operational needs of the ground system can be fulfilled by a very limited number of standard atmospheres selected from Ref. 2.

Derivatives

The function "Derivatives" is performed if the system uses the simple ballistic solution referred to in Table 1 under subparagraph c-1. If the more complex solution (Table 1, subparagraph c-2) is used, this function is omitted.

The inputs to "Derivatives" are values of Time of Fall and Trail, paired with values of Velocity Pressure and Height Pressure selected from the ballistic tables of the Technical Manuals.

As before,

TR_0 and TF_0 are the values of trail and time of fall at the planned release,

and let

ΔVP_0 and ΔHP_0 be the allowable incremental changes in the planned values of VP and HP at release.

Then, select values of TR and TF from the ballistic tables corresponding to $VP_0 \pm \Delta VP_0$ and $HP_0 \pm \Delta HP_0$ and assign index numbers as follows:

	$(VP_0 - \Delta VP_0)$	VP_0	$(VP_0 + \Delta VP)$
$(HP_0 - \Delta HP_0)$	--	TR_1, TF_1	--
HP_0	TR_3, TF_3	TR_0, TF_0	TR_4, TF_4
$(HP_0 + \Delta HP_0)$	--	TR_2, TF_2	--

Then

$$TRH = \frac{TR_2 - TR_1}{2\Delta HP} ,$$

$$TFH = \frac{TF_2 - TF_1}{2\Delta HP} ,$$

$$TRV = \frac{TR_4 - TR_3}{2\Delta VP} ,$$

$$TFV = \frac{TF_4 - TF_3}{2\Delta VP} .$$

The outputs of the function "Derivatives" are TR_0, TF_0, TRH, TFH, TRV, and TFV.

Flight Data

All flight data that do not require processing are transferred directly to the medium by which the Flight Input Data are to be transferred to the Aircraft System. In this instance, the ground computer serves only as a means of ensuring the correct organization of all the data to be transferred to the Aircraft System.

Flight Input Data

The flight input data are the data of Tables 1 and 2. For a specific system these data include only those that pertain to the options embodied in the system. The actual medium used for data transfer to the Aircraft System could be a recording such as magnetic tape; or if there were a requirement for the Ground System and the Aircraft System to be interfaced during flight, all or part of the transfer medium could be a communication link.

5. SUMMARY

This report has described an accurate, four-dimensional aircraft navigation system in terms of the functions performed by various subsystems. Mathematical descriptions of these functions are given in consistent nomenclature. Functions of subsystems of the supporting ground system have been described similarly. In some cases, the functional descriptions have included a number of options, one of which can be chosen to suit the need of a specific system. Also, some of the subsystems may not be required for some systems. For example, all the modes of operation shown in Fig. 3 are not usually a requirement for a single type of aircraft. In such a case the various modes to be deleted are obvious. Deleted aircraft modes or deleted options with respect to certain aircraft subsystems allow the deletion of their ground system counterparts.

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APPENDIX A

RESULTS OF STATISTICAL ANALYSIS OF UDORN MONITOR DATA

The Southeast Asia Loran chain is monitored at Udorn, Thailand. At this site two Loran timers are operated in parallel from the same antenna. The time difference coordinates of the antenna measured by each timer are recorded continuously and used to assess the degree to which the chain is operated within tolerances. From time to time, the U. S. Coast Guard has made these records available for analysis at APL. The complete results are described in the references. The mean and standard deviations, extracted from the references, are listed in the following table. TDX, TDY, and TDZ refer to time differences between the master transmitter and slave transmitters X, Y, and Z, respectively.

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Mean and Standard Deviations of Time Differences at Udorn (in microseconds)
Timer 28*

Period	TDX				TDY				TDZ			
	Bandwidth 4		Bandwidth 3		Bandwidth 4		Bandwidth 3		Bandwidth 4		Bandwidth 3	
	Mean**	Dev	Mean	Dev	Mean	Dev	Mean	Dev	Mean	Dev	Mean	Dev
10-16 Jan '71	2, 663	0, 0158	-	-	2, 130	0, 0200	2, 131	0, 0306	5, 056	0, 0116	-	-
22-25 Apr '70	2, 670	0, 0178	2, 666	0, 0198	2, 119	0, 0188	2, 134	0, 0314	5, 068	0, 0183	5, 041	0, 0233
1-6 Dec '69	2, 664	0, 0190	2, 659	0, 0173	2, 116	0, 0116	2, 110	0, 0257	5, 062	0, 0221	5, 039	0, 0253
29 Aug-6 Sep '69	2, 655	0, 0124	2, 655	0, 0135	2, 102	0, 0094	2, 102	0, 0197	5, 054	0, 0188	5, 039	0, 0229
4-10 May '69	2, 661	0, 0083	2, 659	0, 0107	2, 112	0, 0092	2, 112	0, 0129	-	-	-	-
9-15 Feb '69	2, 659	0, 0144	2, 655	0, 0086	2, 111	0, 0110	2, 108	0, 0146	-	-	-	-
10-15 Nov '68	2, 658	0, 0109	2, 653	0, 0106	2, 105	0, 0107	2, 110	0, 0128	-	-	-	-

Period	TDX All Bandwidths		TDY All Bandwidths	
	Mean	Dev	Mean	Dev
27-29 Oct '68	2, 665	0, 0108	2, 113	0, 0080
24-30 Dec '67	2, 654	0, 0294	2, 103	0, 0339

*Records of Timer 29 are essentially identical. Standard control values are TDX = 12562, 66, TDY = 31132, 11, and TDZ = 43685, 05

**Values are the four least significant digits

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APPENDIX B

MATHEMATICAL DEVELOPMENT OF THE NAVIGATION COORDINATES

Let λ_i, φ_i - latitude and longitude of the i^{th} station, $i = M, A, B$

and λ_O, φ_O - latitude and longitude of the origin,

and Ω_i - central angle from the origin to i^{th} station.

Now the origin of the coordinate system λ_O, φ_O is selected such that $\Omega_M = \Omega_A = \Omega_B$. The solution for λ_O, φ_O is as follows:

$$\cos\Omega_M = \cos\lambda_M \cos\varphi_M \cos\lambda_O \cos\varphi_O + \cos\lambda_M \sin\varphi_M \cos\lambda_O \sin\varphi_O + \sin\lambda_M \sin\lambda_O, \qquad (1)$$

$$\cos\Omega_A = \cos\lambda_A \cos\varphi_A \cos\lambda_O \cos\varphi_O + \cos\lambda_A \sin\varphi_A \cos\lambda_O \sin\varphi_O + \sin\lambda_A \sin\lambda_O, \qquad (2)$$

$$\cos\Omega_B = \cos\lambda_B \cos\varphi_B \cos\lambda_O \cos\varphi_O + \cos\lambda_B \sin\varphi_B \cos\lambda_O \sin\varphi_O + \sin\lambda_B \sin\lambda_O. \qquad (3)$$

Subtracting Eq. (1) from Eq. (2) and dividing by $\cos\lambda_O$,

$$O = (\cos\lambda_A \cos\varphi_A - \cos\lambda_M \cos\varphi_M) \cos\varphi_O + (\cos\lambda_A \sin\varphi_A - \cos\lambda_M \sin\varphi_M) \sin\varphi_O + (\sin\lambda_A - \sin\lambda_M) \tan\lambda_O . \qquad (4)$$

Subtracting Eq. (1) from Eq. (3) and dividing by $\cos\lambda_O$,

$$O = (\cos\lambda_B \cos\varphi_B - \cos\lambda_M \cos\varphi_M) \cos\varphi_O + (\cos\lambda_B \sin\varphi_B - \cos\lambda_M \sin\varphi_M) \sin\varphi_O + (\sin\lambda_B - \sin\lambda_M) \tan\lambda_O . \qquad (5)$$

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Now let

$$a_1 = \cos\lambda_A \cos\varphi_A - \cos\lambda_M \cos\varphi_M,$$
$$a_2 = \cos\lambda_A \sin\varphi_A - \cos\lambda_M \sin\varphi_M,$$
$$a_3 = \sin\lambda_A - \sin\lambda_M,$$
$$b_1 = \cos\lambda_B \cos\varphi_B - \cos\lambda_M \cos\varphi_M,$$
$$b_2 = \cos\lambda_B \sin\varphi_B - \cos\lambda_M \cos\varphi_M,$$
$$b_3 = \sin\lambda_B - \sin\lambda_M.$$

Equations (4) and (5) can be written

$$a_1 \cos\varphi_O + a_2 \sin\varphi_O = - a_3 \tan\lambda_O, \tag{4'}$$
$$b_1 \cos\varphi_O + b_2 \sin\varphi_O = - b_3 \tan\lambda_O. \tag{5'}$$

Now multiplying Eq. (4') by b_3 and Eq. (5') by a_3 and subtracting

$$(a_1 b_3 - b_1 a_3) \cos\varphi_O + (a_2 b_3 - b_2 a_3) \sin\varphi_O = 0. \tag{6}$$

From Eq. (6) it is clear that

$$\varphi_O = \tan^{-1}\left(\frac{b_1 a_3 - b_3 a_1}{a_2 b_3 - b_2 a_3}\right). \tag{7}$$

Now from Eq. (4') we have

$$\lambda_O = \tan^{-1}\left(-\frac{1}{a_3}(a_1 \cos\varphi_O + a_2 \sin\varphi_O)\right). \tag{8}$$

Now that the origin of our system is defined, we shall rotate λ_i, φ_i into this new coordinate system. The cartesian components of λ_i, φ_i are

$$\bar{r}_i = r_e \begin{pmatrix} \cos\lambda_i \cos\varphi_i \\ \cos\lambda_i \sin\varphi_i \\ \sin\lambda_i \end{pmatrix}, \tag{9}$$

where

r_e is the radius of the earth.

Now the components of $\overline{r}_i'$ in our new system are given by

$$\overline{r}_i' = R_2 R_1 \overline{r}_i , \qquad (10)$$

where R_1 is the rotation matrix:

$$\begin{pmatrix} \cos\varphi_O & \sin\varphi_O & O \\ -\sin\varphi_O & \cos\varphi_O & O \\ O & O & 1 \end{pmatrix} , \qquad (11a)$$

and R_2 is the rotation matrix:

$$\begin{pmatrix} \cos\lambda_O & O & \sin\lambda_O \\ O & 1 & O \\ -\sin\lambda_O & O & \cos\lambda_O \end{pmatrix} . \qquad (11b)$$

Now denoting the components of $\overline{r}_i'$ by

$$\overline{r}_i' = \begin{bmatrix} X_i' \\ Y_i' \\ Z_i' \end{bmatrix} . \qquad (12)$$

We now define the pseudo latitude and pseudo longitude of the stations by:

$$\lambda_i' = \tan^{-1}\left(\frac{Z_i'}{\sqrt{X_i'^2 + Y_i'^2}}\right) , \qquad (13)$$

and

$$\varphi_i' = \tan^{-1}\left(\frac{Y_i'}{X_i'}\right) . \qquad (14)$$

Now if the surface of the earth is considered flat, then

$$\theta_i^2 = r_e^2 \left((\lambda_i' - \lambda')^2 + (\varphi_i' - \varphi')^2\right) , \tag{15}$$

where θ_i is the arc length from a point λ', φ' to i^{th} station, $i = M, A, B$. This is equivalent to the small angle approximation

$$\sin\omega = \omega ,$$
$$\cos\omega = 1 - \omega^2/2 \qquad \text{for small } \omega.$$

Our coordinate transformation has assured us of small ω (typically less than 5°). For our case $\omega = \theta/r_e$.

Now letting

$$r_e \lambda_i = Y_i ,$$
$$r_e \varphi_i = X_i ,$$
$$r_e \lambda = Y ,$$
$$r_e \varphi = X .$$

We have for $i = M, A, B$ from Eq. (15),

$$\theta_M^2 = (X_M - X)^2 + (Y_M - Y)^2 , \tag{16}$$

$$\theta_A^2 = (X_A - X)^2 + (Y_A - Y)^2 , \tag{17}$$

$$\theta_B^2 = (X_B - X)^2 + (Y_B - Y)^2 . \tag{18}$$

Equations (17) and (18) can be written as

$$(\theta_A - \theta_M + \theta_M)^2 = (X_A - X)^2 + (Y_A - Y)^2 , \tag{17'}$$

$$(\theta_B - \theta_M + \theta_M)^2 = (X_B - X)^2 + (Y_B - Y)^2 . \qquad (18')$$

Now let

$$\mu_A = \theta_A - \theta_M ,$$
$$\mu_B = \theta_B - \theta_M .$$

Note that our measurements are linear functions of μ_A and μ_B so μ_A and μ_B are inputs. Equations (17') and (18') are

$$\mu_A^2 + 2\mu_A \theta_M + \theta_M^2 = X_A^2 - 2X X_A + X^2$$
$$+ Y_A^2 - 2Y Y_A + Y^2 , \qquad (19)$$

$$\mu_B^2 + 2\mu_A \theta_M + \theta_M^2 = X_B^2 - 2X X_B + X^2$$
$$+ Y_B^2 - 2Y Y_B + Y^2 . \qquad (20)$$

Now subtracting Eq. (16) from Eqs. (19) and (20),

$$\mu_A^2 + 2\mu_A \theta_M = X_A^2 + Y_A^2 + 2(X_M - X_A) X$$
$$+ 2(Y_M - Y_A) Y - (X_M^2 + Y_M^2) , \qquad (21)$$

$$\mu_B^2 + 2\mu_B \theta_M = X_B^2 + Y_B^2 + 2(X_M - X_B) X$$
$$+ 2(Y_M - Y_B) Y - (X_M^2 + Y_M^2) . \qquad (22)$$

Note that in our coordinate system

$$X_M^2 + Y_M^2 = X_A^2 + Y_A^2 = X_B^2 + Y_B^2 ,$$

so Eqs. (21) and (22) can be written

$$\mu_A^2 + 2\mu_A \theta_M = 2(X_M - X_A) X + 2(Y_M - Y_A) Y , \quad (21')$$

$$\mu_B^2 + 2\mu_B \theta_M = 2(X_M - X_B) X + 2(Y_M - Y_B) Y . \quad (22')$$

Let

$$A = Y_M - Y_A ,$$
$$B = Y_M - Y_B ,$$
$$C = X_M - X_A ,$$
$$D = X_M - X_B ,$$

we have

$$AY + CX = 1/2 \, \mu_A^2 + \mu_A \theta_M , \quad (23)$$

$$BY + DX = 1/2 \, \mu_B^2 + \mu_B \theta_M . \quad (24)$$

By Krammer's rule

$$Y = \frac{1}{\Delta}\Big[D(1/2 \, \mu_A^2 + \mu_A \theta_M) - C(1/2 \, \mu_B^2 + \mu_C \theta_M) \Big], \quad (23')$$

$$X = \frac{1}{\Delta}\Big[A(1/2 \, \mu_B^2 + \mu_B \theta_M) - B(1/2 \, \mu_A^2 + \mu_A \theta_M) \Big], \quad (24')$$

where

$$\Delta = A \cdot D - B \cdot C .$$

Now let

$$\alpha = \frac{1}{\Delta}(1/2\ D\mu_A^2 - 1/2\ C\mu_B^2)\,,$$

$$\beta = \frac{1}{\Delta}(D\mu_A - C\mu_B)\,,$$

$$\gamma = \frac{1}{\Delta}(1/2\ A\mu_B^2 - 1/2\ B\mu_A^2)\,,$$

$$\delta = \frac{1}{\Delta}(A\mu_B - B\mu_A)\,.$$

Equations (23') and (24') can be written as

$$Y = \alpha + \beta\theta_M\,, \tag{25}$$

$$X = \gamma + \delta\theta_M\,. \tag{26}$$

Now substituting X, Y from Eqs. (25) and (26) into Eq. (16) gives

$$\theta_M^2 = (X_M - \gamma - \delta\theta_M)^2 + (Y_M - \alpha - \beta\theta_M)^2\,. \tag{27}$$

Expanding Eq. (27),

$$\begin{aligned}\theta_M^2 = {} & Y_M^2 + X_M^2 - 2Y_M\alpha - 2X_M\gamma \\ & - 2Y_M\beta\theta_M - 2X_M\delta\theta_M + \alpha^2 + \gamma^2 \\ & + 2\alpha\beta\theta_M + \beta^2\theta_M^2 + 2\gamma\delta\theta_M + \delta^2\theta_M^2\,.\end{aligned}$$

Collecting terms

$$\begin{aligned}O = {} & (\beta^2 + \delta^2 - 1)\theta_M^2 + 2[\alpha\beta - Y_M\beta + \gamma\delta - X_M\delta]\theta_M \\ & + (Y_M - \alpha)^2 + (X_M - \gamma)^2\,.\end{aligned}$$

Let

$$\eta = \beta^2 + \delta^2 - 1 ,$$

$$\zeta = 2(\alpha\beta - Y_M \beta + \gamma\delta - X_M \delta) ,$$

$$\xi = (Y_M - \alpha)^2 + (X_M - \gamma)^2 . \tag{28}$$

Then

$$\eta\theta_M^2 + \zeta\theta_M + \xi = O .$$

It follows that

$$\theta_M = \frac{-\zeta \pm \sqrt{\zeta^2 - 4\eta\xi}}{2\eta} . \tag{29}$$

Now θ_M from Eq. (29) can be substituted into Eqs. (25) and (26) to give X and Y.

APPENDIX C

DETERMINATION OF TRANSMITTER LOCATIONS IN LORAN RECTANGULAR COORDINATES

Appendix B develops the coordinate transformation that is to be used for aircraft navigation. For simplification of the mathematics, the development is based on a spherical model of the earth. The resulting transformation, referred to as "Loran Rectangular Coordinates," adequately fulfills the requirements for accurate navigation as described in the text of the report. However, complications develop, principally in the vicinity of baseline extensions, if the Loran transmitters are not located in Loran Rectangular Coordinates with sufficient accuracy Analysis of this problem has shown that a spherical earth model is not sufficient for locating the transmitters.

Specifically, the problem is that the locations of the transmitters that would be obtained by the procedures of Appendix B do not constrain the Loran Rectangular Coordinate transformation to exist everywhere for "real-world" measured values of time differences. The mathematics allow imaginary values of X_R and Y_R under some circumstances. The desirable goal of precluding these circumstances may be achieved using "corrected" coordinates of the transmitters and propagation velocity while navigating. The correct values may be obtained by perturbing the transmitter locations and the velocity of propagation in such a way as to match the time differences that identify the six baseline extensions to the real values.

The procedure for doing this is as follows:

1. Obtain published values of the geodetic locations of the transmitters and compute Loran Rectangular Coordinates as Appendix B.

2. Obtain the real baseline lengths in microseconds from calibration data.

Then, using the transmitter locations and baseline lengths, set up the five constraining equations in the seven unknowns. These unknowns are the amounts by which the three pairs of transmitter coordinates and the propagation velocity must be perturbed. Solve these equations for five of the unknowns in terms of the constants and the other two unknowns. Obtain a minimum variance solution for these two unknowns and, using these values, obtain the values of the other five unknowns. The constraining equations are as follows:

$$(X_i - X_j + \Delta X_i - \Delta X_j)^2 + (Y_i - Y_j + \Delta Y_i - \Delta Y_j)^2$$
$$= (V + \Delta V)^2 (L_{i,j})^2 ,$$

where

$i = M, A, B,$

$j = M, A, B,$

$i \neq j$ in the same equation,

X, Y are Loran Rectangular Coordinates

V is the propagation velocity, and

$L_{i,j}$ are the baseline lengths in microseconds.

Permutations of the indices in the equation above result in three equations. Also,

$$(X_i + \Delta X_i)^2 + (Y_i + \Delta Y_i)^2$$
$$= (X_j + \Delta X_j)^2 + (Y_j + \Delta Y_j)^2 .$$

Permutations of the indices result in three of these equations, but one of the three is redundant. Also,

$$\Sigma \Delta X_i^{\ 2} + \Sigma \Delta Y_i^{\ 2} + \Delta V^2 \frac{R_c^{\ 2}}{V^2} = \text{minimum},$$

where $R_c = \sqrt{X_i^2 + Y_i^2}$ using the values obtained in Appendix B. This is the function to be minimized by a least square computational routine.

These equations can be most easily solved by linearizing where appropriate and iterating the solution. Starting values are obtained by Appendix B. Final values are the "corrected" values to be used in the navigation equations.

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